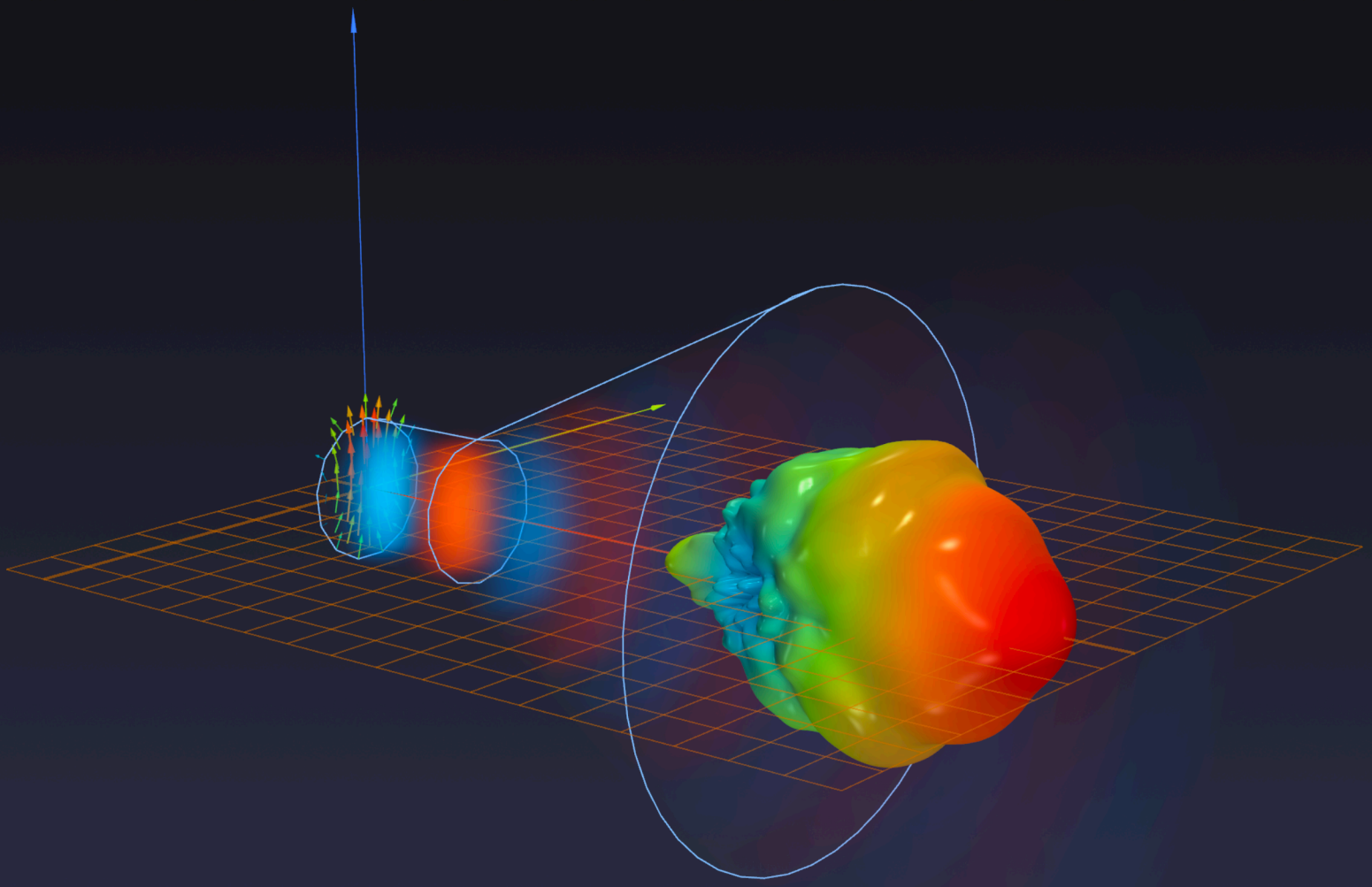
EMerge Software



VALIDATION & VERIFICATION

Solver Validation Report

Benchmarking the **EMerge** full-wave FEM solver against analytical references

SOFTWARE	DOCUMENT	DATE	RESULT
EMerge 3.0.0	EMS-VAL-3.0.0	04 November 2026	8 / 8 cases passed

DOCUMENT CONTROL

EMerge Solver Validation Report

Document number	EMS-VAL-3.0.0
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Validation cases	8 (A1, A2, A3, A4, A5, A6, A7, A8)
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REVISION HISTORY

Rev.	Date	Description
A	04 November 2026	First issue. Analytical cases A1–A8.

> **IMPORTANT**
>
> The results in this report were generated automatically from the validation scripts in the `cases/` folder. Every number, table and figure is read from the `results/` folder at compile time. A result is only valid for the software version and test environment stated in Section 2.5.

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1. Introduction

1.1 Purpose

This report documents the validation of the EMerge finite-element solver for time-harmonic electromagnetic problems. Each validation case compares EMerge results with an independent reference under fixed, scripted conditions and grades the outcome against acceptance criteria that are stated before the comparison. The report is regenerated for every release so that regressions in accuracy are found early.

1.2 Scope

The report covers the 3D frequency-domain microwave solver of EMerge 3.0.0: the port eigenmode solver, port boundary conditions, the Nédélec volume discretisation, the linear solvers and the post-processing of S-parameters and fields. Solver performance is recorded for information only; it is not graded.

1.3 Test categories

The validation cases are grouped in four categories of increasing difficulty and decreasing certainty of the reference.

Category	Description
Analytical	Problems with an exact closed-form or quasi-exact semi-analytical solution. They are simple by design and check that the fundamental kernels are exact and free of defects.
Cross-validation	Problems whose reference comes from an independent, established solver, converged to high accuracy.
Literature	Measured structures from the literature. Manufacturing tolerances preclude an exact match; the goal is agreement within engineering accuracy.
Pathological	Problems designed to stress a solver: singularities, extreme aspect ratios, ill-conditioning. They identify the limits of the method.

2. Methodology

2.1 Validation procedure

Every case is implemented as a self-contained Python script in `cases/`. A script builds the EMerge model, runs one or more studies, computes the reference solution, evaluates the acceptance criteria and writes its results to `results/<ID>/`:

- `summary.json`: parameters, settings, solver statistics, criteria and verdict;
- `*.csv`: the raw data of every study (S-parameters, refinement sequences);
- `*.svg`, `*.png`: the figures reproduced in this report.

This document reads those files at compile time; nothing is transcribed by hand. A case typically consists of a nominal frequency sweep compared against the reference, complemented where relevant by a refinement study that demonstrates convergence towards it.

2.2 Conventions

All quantities use the $e^{j\omega t}$ time convention, so a wave travelling in $+y$ varies as $e^{-j\beta y}$. S-parameters are normalised to the modal impedance of the port modes, and reference planes coincide with the port faces unless stated otherwise. The mesh resolution is specified as the maximum element size in fractions of the free-space wavelength λ_0 at the highest simulated frequency.

2.3 Error metrics

Each case is judged on two criteria that together show conclusively that the model works; typically one on the reflection and one on the transmission. For a computed scattering parameter S and its reference S^{ref} the following metrics are used.

Metric	Definition	Interpretation
Complex error	$\lvert S - S^{\mathrm{ref}}\rvert$	Vector distance in the complex plane; captures magnitude and phase. Expressed in dB.
Phase error	$\angle\left(S/S^{\mathrm{ref}}\right)$	Wrapped phase difference in degrees; sensitive to propagation constant and length.
Relative error	$\lvert X - X^{\mathrm{ref}}\rvert \;/\; \lvert X^{\mathrm{ref}}\rvert$	For scalar quantities such as the characteristic impedance; in percent.

2.4 Acceptance criteria and verdict

Each case defines numbered criteria with a fixed limit. Limits are set at a level that an engineering user would consider fully adequate (for instance −40 dB residual reflection, 0.5° phase error) and are not tuned to the result obtained. A criterion passes when its metric meets the limit. A case passes when **all** of its criteria pass.

2.5 Test environment

TABLE 1 Environment in which the results of this report were produced.

Item	Value
EMerge version	3.0.0
Python / NumPy / SciPy	3.11.14 / 2.2.6 / 1.17.1
Operating system	Darwin 25.5.0 (arm64)
Processor	Apple M3 Ultra (28 logical cores)
Memory	256 GB

3. SUMMARY OF RESULTS

8 / 8	18 / 18	309.0 s	3.0.0
CASES PASSED	CRITERIA MET	TOTAL RUN TIME	EMERGE VERSION

TABLE 2 Compliance matrix: overall verdict of every validation case in this report.

Case	Title	Category	Criteria	Verdict
A1	Straight rectangular waveguide	Analytical	2 / 2	PASS
A2	Short-circuited waveguide	Analytical	2 / 2	PASS
A3	Waveguide with dielectric block	Analytical	2 / 2	PASS
A4	Partially loaded waveguide	Analytical	2 / 2	PASS
A5	Stripline characteristic impedance	Analytical	2 / 2	PASS
A6	Capacitive iris	Analytical	4 / 4	PASS
A7	Inductive iris	Analytical	2 / 2	PASS
A8	Capacitive and inductive iris in series	Analytical	2 / 2	PASS

4. ANALYTICAL TESTS

This chapter contains the cases with an exact closed-form or quasi-exact semi-analytical reference. Cases A1–A4 are hollow or dielectric-loaded waveguides that test ports, materials and propagation. Case A5 tests the port eigenmode solver on a TEM line. Cases A6–A8 test irises against converged mode-matching solutions (Appendix A): infinitely thin septa with singular edges (A6, A7), and two finite-thickness irises in series (A8).

4.1 A1: Straight rectangular waveguide

CASE	CATEGORY	CRITERIA MET	VERDICT
A1	Analytical	2 / 2	PASS

Script: `cases/A1_straight_waveguide.py` Run: 2026-11-04 16:17:36 · EMerge 3.0.0 · 4.9 s

Hollow PEC rectangular waveguide terminated by a TE10 waveguide port at each end. Tests the port eigenmode, the port boundary condition and the propagation of the fundamental mode against the closed-form solution. An error at this level points to a defect in the fundamental kernels, not to modelling difficulty.

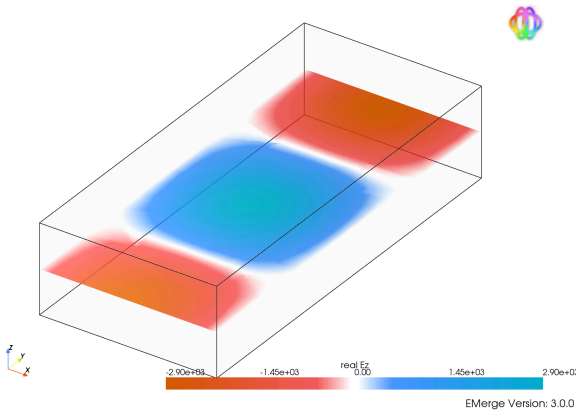


FIGURE 1 Real part of E_z on the mid-height plane at 9 GHz.

TABLE 3 Model parameters of case A1.

Parameter	Symbol	Value	Unit
Broad wall width	a	23	mm
Narrow wall height	b	11.5	mm
Length	L	50	mm
TE10 cut-off frequency	f_c	6.517	GHz
Frequency range	f	8 – 10	GHz

Reference solution

The TE₁₀ mode propagates with $\beta = \sqrt{k_0^2 - (\pi/a)^2}$. A uniform, lossless guide neither reflects nor attenuates, so

$$S_{11} = S_{22} = 0, \qquad S_{21} = S_{12} = e^{-j\beta L}. \tag{1}$$

Results

896	7 128	41	0.41 s
TETRAHEDRA	DEGREES OF FREEDOM	FREQUENCY POINTS	TOTAL SOLVE TIME

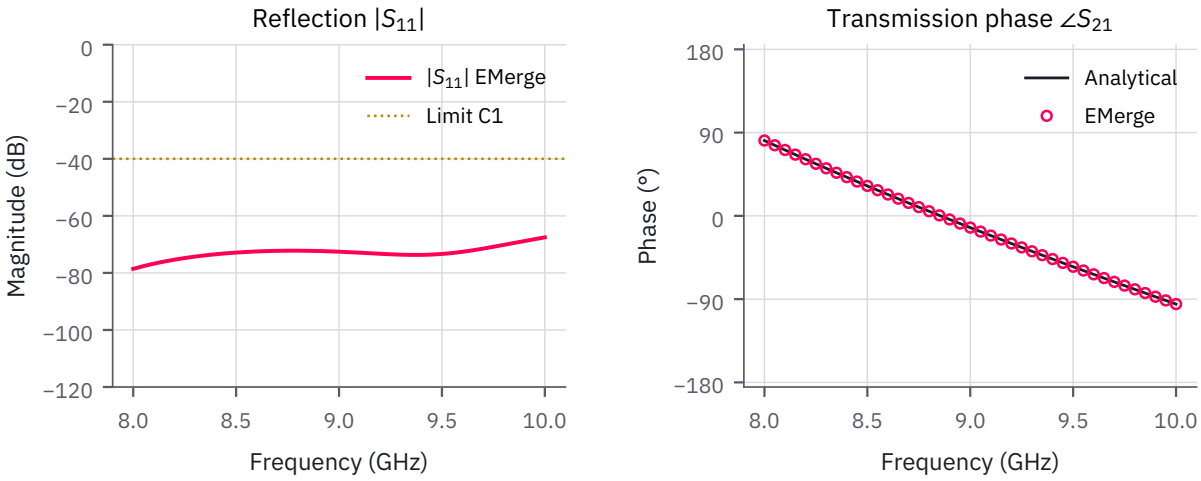


FIGURE 2 Residual reflection (left) and transmission phase (right) against the analytical solution.

TABLE 4 Acceptance criteria for case A1 and their outcome.

ID	Quantity	Metric	Result	Limit	Verdict
C1	Reflection $\lvert S_{11}\rvert, \lvert S_{22}\rvert$	max over band	−67.5 dB	≤ −40.0 dB	PASS
C2	Transmission phase error $\lvert\angle S_{21} - \angle S_{21}^{\mathrm{ref}}\rvert$	max over band	0.039°	≤ 0.500°	PASS

Mesh refinement

Figure 3 shows that the agreement is a converged property of the discretisation and not a coincidence of one mesh. Over the 4 finest meshes the transmission error decreases as h^p with $p = 3.5$, and it reaches −92 dB at 97 102 degrees of freedom.

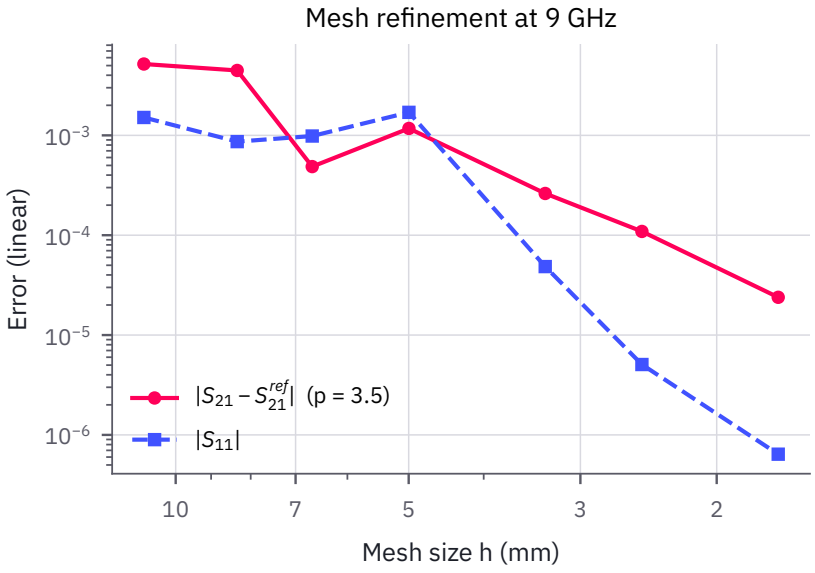


FIGURE 3 Errors at 9 GHz under uniform mesh refinement.

CONCLUSION — CASE A1 PASS

EMerge reproduces the exact scattering matrix of the straight waveguide over the full band and converges systematically under mesh refinement.

4.2 A2: Short-circuited waveguide

CASE	CATEGORY	CRITERIA MET	VERDICT
A2	Analytical	2 / 2	PASS

Script: `cases/A2_shorted_waveguide.py` Run: 2026-11-04 16:17:39 · EMerge 3.0.0 · 1.9 s

The straight waveguide of A1 terminated in a PEC short circuit. Tests total reflection and the round-trip phase of a one-port network. Port 2 of case A1 is removed, so the back face keeps the default PEC condition and the guide becomes a one-port network.

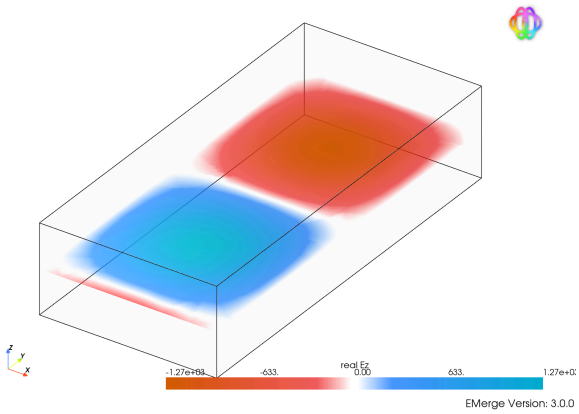


FIGURE 4 Standing wave ($\Re\, E_z$, mid-height plane) in front of the short at 9 GHz.

TABLE 5 Model parameters of case A2.

Parameter	Symbol	Value	Unit
Broad wall width	a	23	mm
Narrow wall height	b	11.5	mm
Port-to-short distance	L	50	mm
Frequency range	f	8 – 10	GHz

Reference solution

The wave travels to the short and back ($2L$) and is inverted on reflection:

$$S_{11} = -e^{-j2\beta L}, \qquad |S_{11}| = 1. \tag{2}$$

Results

896	7 128	41	0.37 s
TETRAHEDRA	DEGREES OF FREEDOM	FREQUENCY POINTS	TOTAL SOLVE TIME

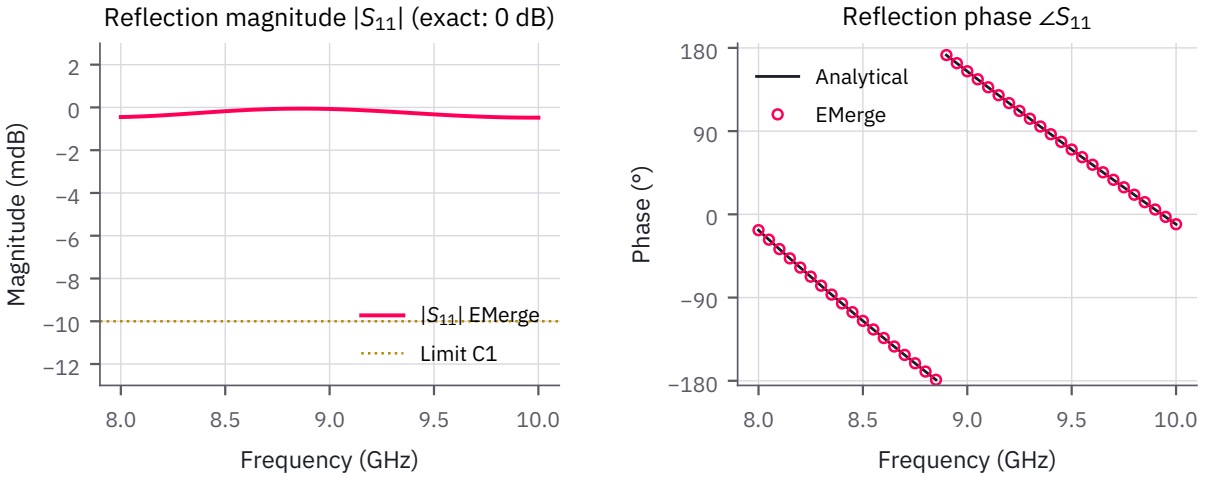


FIGURE 5 Reflection magnitude in milli-decibel (left) and reflection phase (right) against the analytical solution.

TABLE 6 Acceptance criteria for case A2 and their outcome.

ID	Quantity	Metric	Result	Limit	Verdict
C1	Reflection magnitude deviation $\|20\log_{10}\|S_{11}\|\|$	max over band	4.79×10^{-4} dB	≤ 1.00×10^{-2} dB	PASS
C2	Reflection phase error $\|\angle S_{11} - \angle S_{11}^{\mathrm{ref}}\|$	max over band	0.093°	≤ 0.500°	PASS

CONCLUSION — CASE A2 PASS

The PEC termination reflects all incident power, and the round-trip phase matches the analytical value over the full band.

4.3 A3: Waveguide with dielectric block

CASE	CATEGORY	CRITERIA MET	VERDICT
A3	Analytical	2 / 2	PASS

Script: `cases/A3_dielectric_block.py` Run: 2026-11-04 16:17:58 · EMerge 3.0.0 · 18.0 s

A dielectric block filling the full cross-section is centred in the straight waveguide. Tests material interfaces and resonant multiple reflections against the closed-form Fabry–Pérot solution. The block is ℓ long and centred, leaving $(L-\ell)/2$ of empty guide on either side.

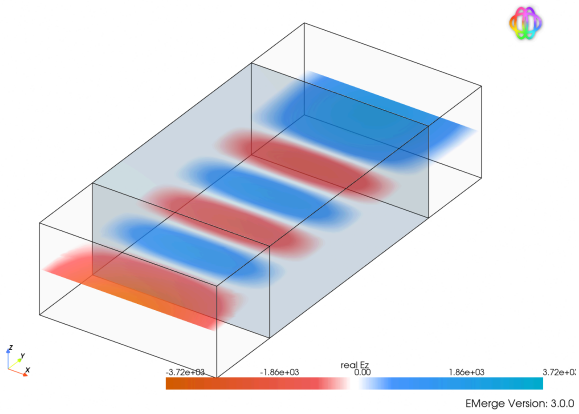


FIGURE 6 $\Re\, E_z$ at 9 GHz: the guided wavelength shortens inside the block.

TABLE 7 Model parameters of case A3.

Parameter	Symbol	Value	Unit
Broad wall width	a	23	mm
Narrow wall height	b	11.5	mm
Port-to-port length	L	50	mm
Block length	ℓ	30	mm
Block permittivity	ε_r	6	
Frequency range	f	8 – 10	GHz

Reference solution

For TE$_{10}$ each face of the block behaves as a junction between two transmission lines with propagation constants β (air) and $\beta_d = \sqrt{\varepsilon_r k_0^2 - (\pi/a)^2}$ (dielectric). The interface reflection and transmission coefficients are

$$r_1 = \frac{\beta - \beta_d}{\beta + \beta_d}, \quad r_2 = -r_1, \quad t_{1,2} = 1 + r_{1,2}. \tag{3}$$

The infinite sum of internal reflections, with $\psi = e^{-j\beta_d \ell}$, converges to

$$S_{11} = e^{-j\beta(L-\ell)}\left(r_1 + \frac{t_1 t_2 r_2 \psi^2}{1 - r_2^2 \psi^2}\right), \quad S_{21} = e^{-j\beta(L-\ell)}\frac{t_1 t_2 \psi}{1 - r_2^2 \psi^2}. \tag{4}$$

This is identical to the Fresnel formulation of the model report, because $n \cos\theta_t = \beta_d / k_0$.

Results

8 005	55 776	41	15.29 s
TETRAHEDRA	DEGREES OF FREEDOM	FREQUENCY POINTS	TOTAL SOLVE TIME

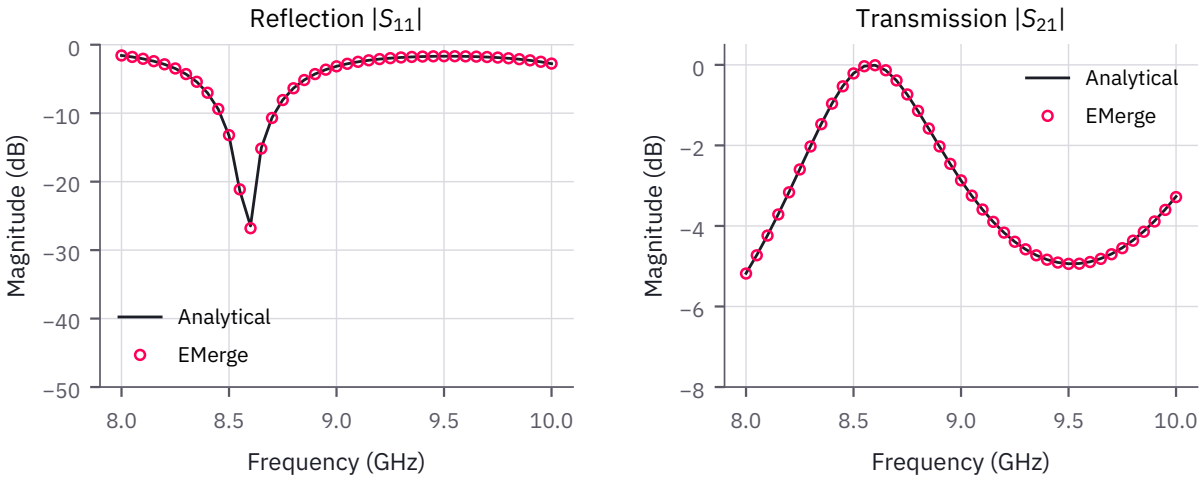


FIGURE 7 Reflection (left) and transmission (right) of the dielectric block against the analytical solution. The reflection zero near 8.6 GHz is the half-wave resonance of the block.

TABLE 8 Acceptance criteria for case A3 and their outcome.

ID	Quantity	Metric	Result	Limit	Verdict
C1	Reflection error $\lvert S_{11} - S_{11}^{\mathrm{ref}} \rvert$	max over band	−54.5 dB	≤ −50.0 dB	PASS
C2	Transmission error $\lvert S_{21} - S_{21}^{\mathrm{ref}} \rvert$	max over band	−53.4 dB	≤ −50.0 dB	PASS

CONCLUSION — CASE A3 PASS

The material interfaces and the resonant multiple reflections are reproduced, including the depth and position of the reflection zero.

4.4 A4: Partially loaded waveguide

CASE	CATEGORY	CRITERIA MET	VERDICT
A4	Analytical	2 / 2	PASS

Script: `cases/A4_partially_loaded_waveguide.py` Run: 2026-11-04 16:18:07 · EMerge 3.0.0 · 7.7 s

A thin dielectric slab is placed along the full length of the guide, centred on the broad wall. The mode is no longer TE10 and its propagation constant follows from a transcendental equation. Tests the modal port solver on an inhomogeneous cross-section. The ports are generic modal ports: their mode profile and propagation constant are computed by the eigenmode solver, not prescribed.

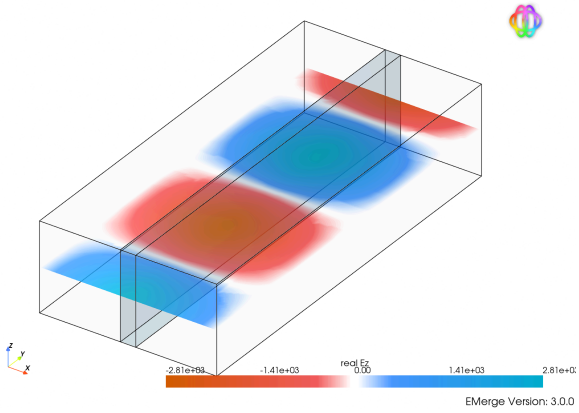


FIGURE 8 $\Re\, E_z$ at 9 GHz. The slab runs along the full length at the centre of the broad wall.

TABLE 9 Model parameters of case A4.

Parameter	Symbol	Value	Unit
Broad wall width	a	23	mm
Narrow wall height	b	11.5	mm
Length	L	50	mm
Slab thickness	t	2	mm
Slab permittivity	ε_r	3	
Frequency range	f	8 – 10	GHz

Reference solution

In the air (1) and slab (2) regions $k_{x1}^2 + \beta^2 = k_0^2$ and $k_{x2}^2 + \beta^2 = \varepsilon_r k_0^2$. Matching E_z and H_y at both interfaces gives, for the mode that is even about the centre line,

$$k_{x1} \cot\left(k_{x1}\frac{a-t}{2}\right) = k_{x2} \tan\left(k_{x2}\frac{t}{2}\right). \tag{5}$$

It has no closed form. It is solved with Brent's method to a tolerance of 10^{-14}, which gives β and the reference $S_{21} = e^{-j\beta L}$ with $S_{11} = 0$. Compared with the empty guide, the slab adds 128° to 138° of transmission phase over the band. A solver that ignored the slab would therefore fail C1 by a wide margin.

Results

2 666	19 184	41	2.61 s
TETRAHEDRA	DEGREES OF FREEDOM	FREQUENCY POINTS	TOTAL SOLVE TIME

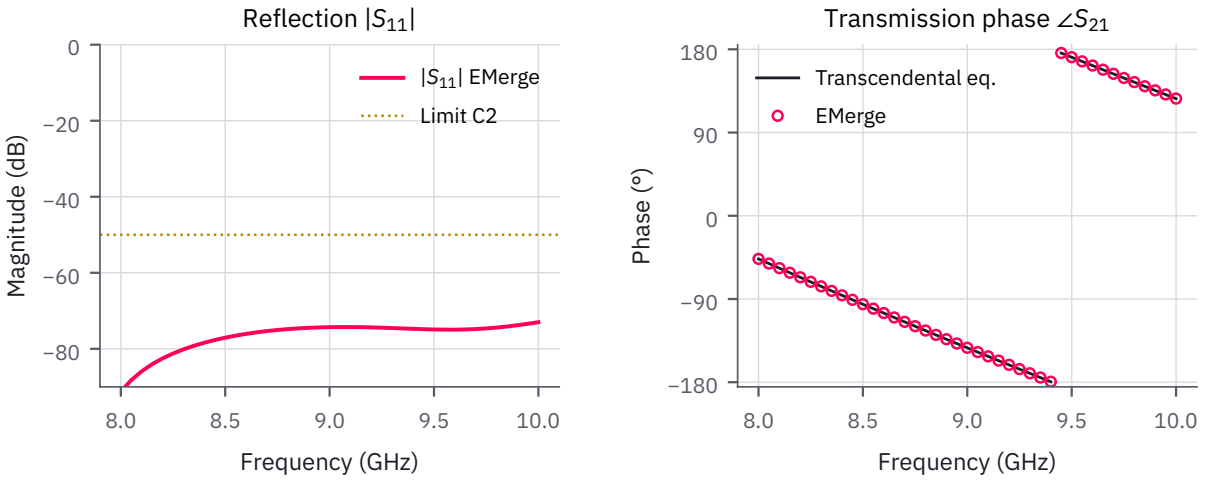


FIGURE 9 Residual reflection (left) and transmission phase (right) against the root of the transcendental equation.

TABLE 10 Acceptance criteria for case A4 and their outcome.

ID	Quantity	Metric	Result	Limit	Verdict
C1	Transmission phase error $\lvert\angle S_{21} - \angle S_{21}^{\mathrm{ref}}\rvert$	max over band	0.089°	≤ 0.100°	PASS
C2	Reflection $\lvert S_{11}\rvert, \lvert S_{22}\rvert$	max over band	−73.0 dB	≤ −50.0 dB	PASS

CONCLUSION — CASE A4 PASS

The modal port solver finds the correct mode of the inhomogeneous cross-section, and the modal ports absorb it without measurable reflection.

4.5 A5: Stripline characteristic impedance

CASE	CATEGORY	CRITERIA MET	VERDICT
A5	Analytical	2 / 2	PASS

Script: `cases/A5_stripline.py` Run: 2026-11-04 16:18:22 · EMerge 3.0.0 · 14.8 s

A zero-thickness strip centred between two ground planes in a homogeneous dielectric. The port eigenmode solver computes the TEM mode of the cross-section; its characteristic impedance and effective index are compared with Cohn's exact conformal-mapping solution over a 25:1 range of strip widths.

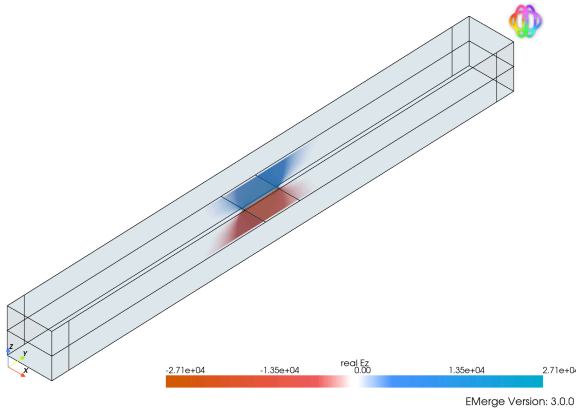


FIGURE 10 $\Re E_z$ on a cross-section of a $\approx 50\,\Omega$ line at 3 GHz: the field reverses sign across the strip, as expected for a stripline TEM mode.

TABLE 11 Model parameters of case A5.

Parameter	Symbol	Value	Unit
Ground plane spacing	b	2	mm
Permittivity	ε_r	2.1	
Strip width range	W	0.2 – 5 (15 values)	mm
Frequency	f	3	GHz

Reference solution

Conformal mapping of the zero-thickness, centred stripline (Cohn, 1954) gives the exact result

$$Z_0 = \frac{30\pi}{\sqrt{\varepsilon_r}} \frac{K(k)}{K(k')}, \quad k = \frac{1}{\cosh(\pi W/2b)}, \quad k' = \tanh(\pi W/2b), \tag{6}$$

where K is the complete elliptic integral of the first kind. Wheeler's design formula is shown for comparison only: it is itself in error by up to 1.3 %. A homogeneously filled line is pure TEM, so $n_{\text{eff}} = \sqrt{\varepsilon_r}$ for every width. EMerge computes Z_0 from the mode power and voltage ($Z_0 = |V|^2 \,/2P$).

Results

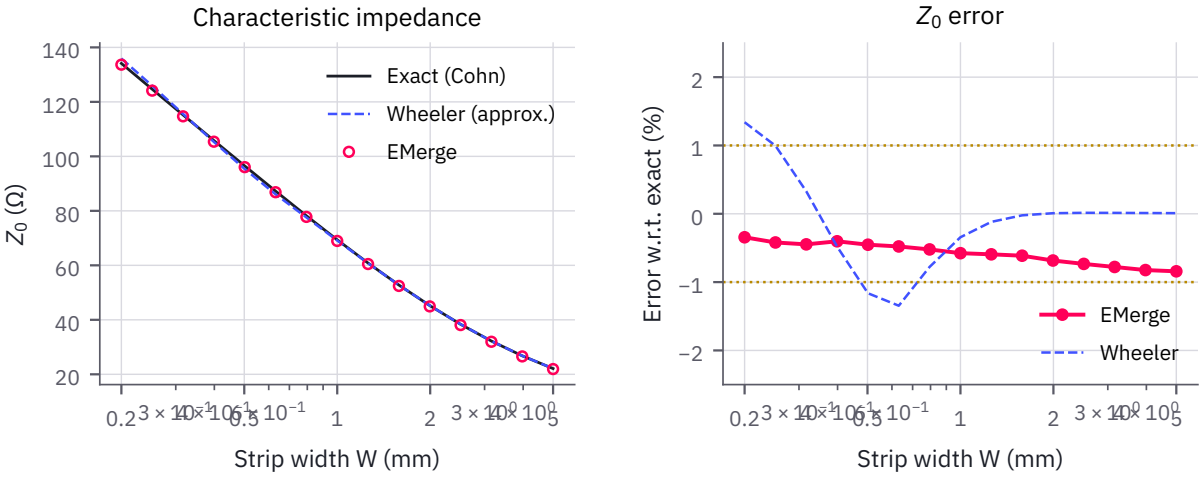


FIGURE 11 Characteristic impedance against strip width (left) and its relative error (right). Dotted lines: acceptance limit C1.

TABLE 12 Acceptance criteria for case A5 and their outcome.

ID	Quantity	Metric	Result	Limit	Verdict
C1	Impedance error $\lvert Z_0 - Z_0^{\text{exact}}\rvert \,/\, Z_0^{\text{exact}}$	max over widths	0.84 %	≤ 1.00 %	PASS
C2	Effective index error $\lvert n_{\text{eff}} - \sqrt{\varepsilon_r}\rvert \,/\, \sqrt{\varepsilon_r}$	max over widths	0.000 %	≤ 0.100 %	PASS

Edge refinement

The remaining impedance error is a discretisation error at the strip edges, where the field is singular. Refining the edge mesh of the $W = 1$ mm line halves the error with every halving of the edge element size (Table 13). This first-order convergence is expected for a field singularity and confirms that the result converges to Cohn's solution.

TABLE 13 Edge mesh refinement at $W = 1$ mm.

Edge element size (µm)	Z_0 (Ω)	Error (%)
200	67.994	−1.96
100	68.616	−1.06
50	68.953	−0.58
25	69.129	−0.32

CONCLUSION — CASE A5 PASS

The port eigenmode solver reproduces the exact stripline impedance to within 0.84 % from 22 to 134 Ω, and the effective index of the TEM mode exactly. The impedance converges towards the exact value as the strip edges are refined.

4.6 A6: Capacitive iris

CASE	CATEGORY	CRITERIA MET	VERDICT
A6	Analytical	4 / 4	PASS

Script: `cases/A6_capacitive_iris.py` Run: 2026-11-04 16:20:57 · EMerge 3.0.0 · 153.6 s

Two infinitely thin PEC septa leave a centred opening of height w across the full width of the guide. The iris acts as a shunt capacitance; the reference is a converged mode-matching solution. Tests sharp PEC edges, with both incomplete and complete second-order bases on one identical mesh. Both bases are solved on one identical, fixed mesh: 0.08 λ_0 in the volume and 150 µm at the inner septum edges. The difference between the two runs is therefore due to the basis alone.

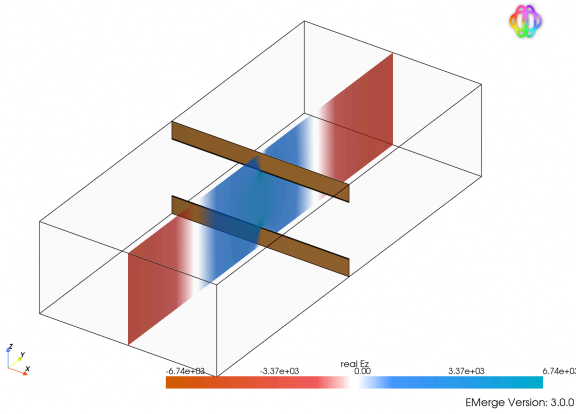


FIGURE 12 $\Re\, E_z$ at 9 GHz on the vertical centre plane (incomplete basis). The septa leave a 7 mm opening.

TABLE 14 Model parameters of case A6.

Parameter	Symbol	Value	Unit
Broad wall width	a	23	mm
Narrow wall height	b	11.5	mm
Port-to-port length	L	50	mm
Iris opening	w	7	mm
Frequency range	f	8 – 10	GHz

Reference solution

The reference is a mode-matching solution (Appendix A). The septa change the guide height, so the incident TE_{10} mode couples to the LSE_{1n} modes of the guide (even n by symmetry). The zero-thickness iris is solved to convergence and is then exactly a shunt admittance $Y = jB$ in the middle of the guide:

$$S_{11} = -\frac{jB}{2 + jB} e^{-j\beta L}, \quad S_{21} = \frac{2}{2 + jB} e^{-j\beta L}. \tag{7}$$

For a lossless shunt element the susceptance also follows from the reflection magnitude alone, $B = 2\,|S_{11}|\,/\sqrt{1 - |S_{11}|^2}$. This isolates the iris from the propagation phase in the uniform guide sections.

Results

TABLE 15 Both bases on the identical mesh, 11 frequency points from 8 – 10 GHz.

Basis	Tetrahedra	DOF	Solve (s)	B error, 8 → 10 GHz	Max. $\lvert\Delta S_{11}\rvert$	Max. $\lvert\Delta S_{21}\rvert$
Incomplete 2nd order (Webb)	40 966	265 354	33.6	2.77 % … 2.26 %	−51.4 dB	−51.7 dB
Complete 2nd order	40 966	398 031	112.3	−0.44 % … 0.36 %	−67.4 dB	−66.2 dB

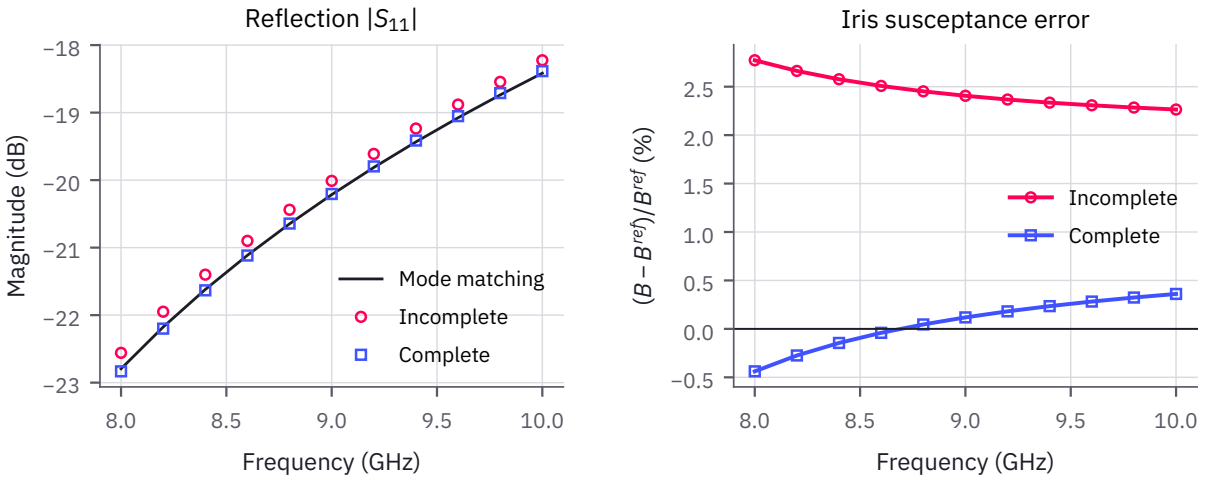


FIGURE 13 Reflection magnitude (left) and iris susceptance error (right) of both bases against the mode-matching reference.

TABLE 16 Acceptance criteria for case A6 and their outcome.

ID	Quantity	Metric	Result	Limit	Verdict
C1	Reflection error $\lvert S_{11} - S_{11}^{\mathrm{ref}} \rvert$, incomplete basis	max over band	−51.4 dB	≤ −50.0 dB	PASS
C2	Transmission error $\lvert S_{21} - S_{21}^{\mathrm{ref}} \rvert$, incomplete basis	max over band	−51.7 dB	≤ −50.0 dB	PASS
C3	Reflection error $\lvert S_{11} - S_{11}^{\mathrm{ref}} \rvert$, complete basis	max over band	−67.4 dB	≤ −50.0 dB	PASS
C4	Transmission error $\lvert S_{21} - S_{21}^{\mathrm{ref}} \rvert$, complete basis	max over band	−66.2 dB	≤ −50.0 dB	PASS

CONCLUSION — CASE A6 PASS

Both bases reproduce the mode-matching reference to better than −50 dB on the same fixed, edge-refined mesh. The complete basis is the more accurate by about 16 dB, at 1.5 times the number of unknowns.

4.7 A7: Inductive iris

CASE	CATEGORY	CRITERIA MET	VERDICT
A7	Analytical	2 / 2	PASS

Script: `cases/A7_inductive_iris.py` Run: 2026-11-04 16:21:19 · EMerge 3.0.0 · 20.5 s

Two infinitely thin PEC septa leave a centred opening of width w along the broad wall. The iris acts as a shunt inductance; the reference is a converged mode-matching solution. The edge singularity lies in the transverse plane, which makes this the most demanding discretisation test of the chapter. The TE_{10} field and the septa are uniform along the narrow wall, so the guide height is reduced to $b = 0.05a$. This leaves the reference unchanged but keeps the edge-refined mesh small.

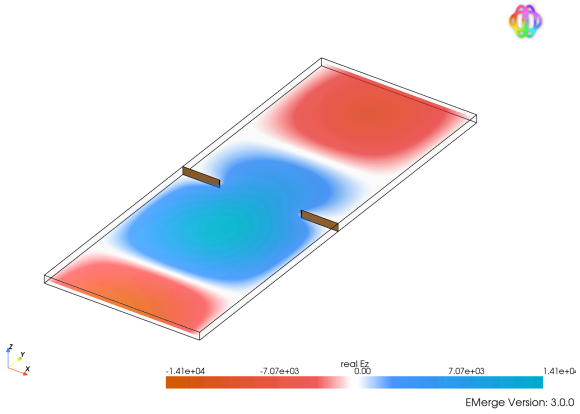


FIGURE 14 $\Re E_z$ at 9 GHz on the mid-height plane. The field concentrates in the 12 mm opening between the septa.

TABLE 17 Model parameters of case A7.

Parameter	Symbol	Value	Unit
Broad wall width	a	23	mm
Narrow wall height	b	1.15	mm
Port-to-port length	L	60	mm
Iris opening	w	12	mm
Frequency range	f	8 – 10	GHz

Reference solution

The reference is a mode-matching solution (Appendix A). The septa change the guide width, so the incident TE_{10} mode couples only to the odd $TE_{n\,0}$ modes. The zero-thickness iris is solved to convergence and shifted to the port planes:

$$S_{11} = S_{11}^{\text{iris}} e^{-j\beta L}, \quad S_{21} = S_{21}^{\text{iris}} e^{-j\beta L}. \tag{8}$$

At zero thickness the converged result depends on the ratio of aperture to guide modes; it is correct only for the ratio w/a used here (relative convergence). The classic variational formula of Rozzi et al. [1] is about 1 % low in susceptance for this iris, which alone corresponds to a complex S-parameter error of about −46 dB.

Results

26 451	175 340	11	17.35 s
TETRAHEDRA	DEGREES OF FREEDOM	FREQUENCY POINTS	TOTAL SOLVE TIME

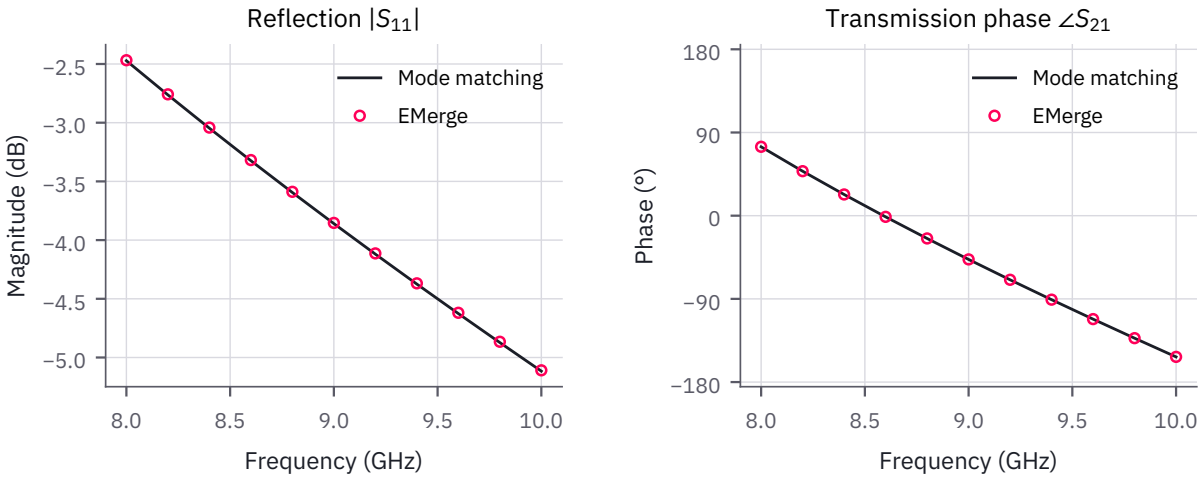


FIGURE 15 Reflection magnitude (left) and transmission phase (right) against the mode-matching reference.

TABLE 18 Acceptance criteria for case A7 and their outcome.

ID	Quantity	Metric	Result	Limit	Verdict
C1	Reflection error $\lvert S_{11} - S_{11}^{\mathrm{ref}} \rvert$	max over band	−64.3 dB	≤ −60.0 dB	PASS
C2	Transmission error $\lvert S_{21} - S_{21}^{\mathrm{ref}} \rvert$	max over band	−64.1 dB	≤ −60.0 dB	PASS

Convergence at the septum edges

NOTE

Near the edge of an infinitely thin septum, $\boldsymbol{H} = (j\omega\mu)^{-1}\nabla \times \boldsymbol{E}$ is singular, and here the singular edges lie across the direction of the incident field. Polynomial Nédélec elements resolve this with local refinement (20 µm at the septum edges, growth 2) and a 0.08 λ_0 volume mesh.

CONCLUSION — CASE A7 PASS

With local refinement at the septum edges, EMerge agrees with the converged mode-matching solution to better than −64.3 dB over the band, despite the edge singularity.

4.8 A8: Capacitive and inductive iris in series

CASE	CATEGORY	CRITERIA MET	VERDICT
A8	Analytical	2 / 2	PASS

Script: `cases/A8_two_irises.py` Run: 2026-11-04 16:22:48 · EMerge 3.0.0 · 87.6 s

A capacitive and an inductive iris of finite thickness, a short distance apart in the same guide. Tests finite-thickness PEC objects and the interaction of two discontinuities against a cascade of exact mode-matching solutions. Unlike A6 and A7 the septa have a finite thickness of 0.5 mm, and the structure is not symmetric, so S_{11} and S_{22} differ in phase.

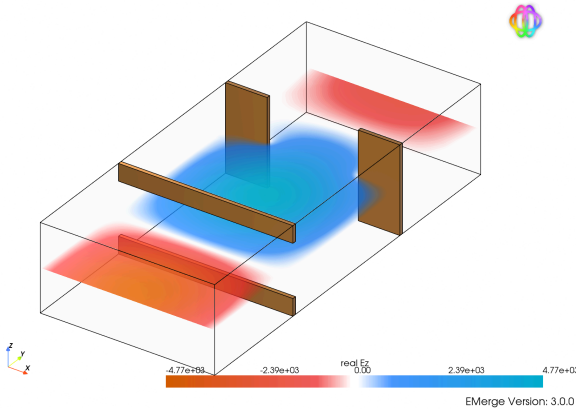


FIGURE 16 $\Re E_z$ at 9 GHz on the mid-height plane. The capacitive iris (horizontal septa) is followed by the inductive iris (vertical septa) 20 mm further on.

TABLE 19 Model parameters of case A8.

Parameter	Symbol	Value	Unit
Broad wall width	a	23	mm
Narrow wall height	b	11.5	mm
Iris thickness	t	0.5	mm
Capacitive opening	w_c	7	mm
Inductive opening	w_i	12	mm
Iris spacing	d	20	mm
Port-to-port length	L	51	mm
Frequency range	f	8 – 10	GHz

Reference solution

Each iris is solved separately by mode matching for its finite thickness (Appendix A). Both are symmetric, so each is described by (S^i_{11}, S^i_{21}) at its own faces. The two are cascaded through the fundamental mode over the spacing d:

$$S_{11} = S^c_{11} + \frac{(S^c_{21})^2 S^i_{11} e^{-2j\beta d}}{1 - S^c_{11} S^i_{11} e^{-2j\beta d}}, \quad S_{21} = \frac{S^c_{21} S^i_{21} e^{-j\beta d}}{1 - S^c_{11} S^i_{11} e^{-2j\beta d}}, \tag{9}$$

with S_{22} following by exchanging the irises. All three are shifted to the port planes.

The cascade neglects the coupling of evanescent higher-order modes between the irises. The slowest-decaying of these modes is attenuated by −61 dB over the spacing. Moreover, it cannot couple back into the fundamental mode at the other iris. The inductive iris is uniform along the height, so it does not scatter the height-dependent LSE_{12} mode of the capacitive iris into TE_{10}. Likewise, the capacitive iris is uniform across the width and does not scatter the TE_{30} mode of the inductive iris into TE_{10}. The neglected coupling therefore lies far below the acceptance limit.

Results

73 181	472 910	11	81.80 s
TETRAHEDRA	DEGREES OF FREEDOM	FREQUENCY POINTS	TOTAL SOLVE TIME

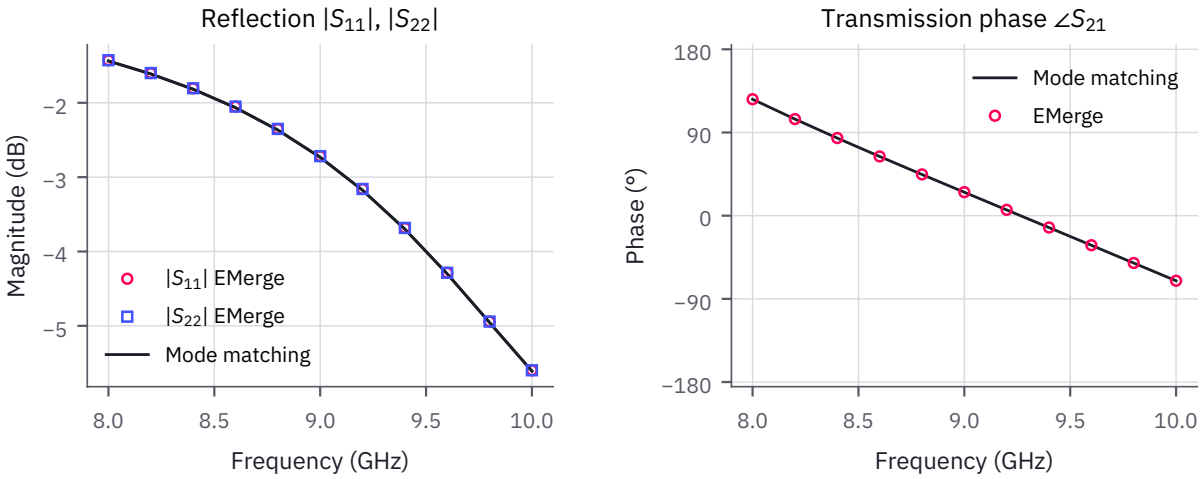


FIGURE 17 Reflection magnitudes (left) and transmission phase (right) against the cascaded mode-matching reference. For a lossless two-port $|S_{11}| = |S_{22}|$; the two reflections differ in phase only, which the complex-error criterion includes.

TABLE 20 Acceptance criteria for case A8 and their outcome.

ID	Quantity	Metric	Result	Limit	Verdict
C1	Reflection error $\lvert S_{11} - S_{11}^{\mathrm{ref}}\rvert, \lvert S_{22} - S_{22}^{\mathrm{ref}}\rvert$	max over band	−54.7 dB	≤ −50.0 dB	PASS
C2	Transmission error $\lvert S_{21} - S_{21}^{\mathrm{ref}}\rvert$	max over band	−54.8 dB	≤ −50.0 dB	PASS

CONCLUSION — CASE A8 PASS

EMerge reproduces the scattering matrix of two different finite-thickness discontinuities in series, including the asymmetric reflection phases, to better than −54.7 dB.

REFERENCES

[1] T. Rozzi, A. Morini, F. Ragusini, and M. Mongiardo, “Analytical Solution and Frequency Extraction of Iris Problems in Waveguide by Separation of Variables,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 45, no. 2, pp. 253–259, Feb. 1997.

APPENDIX A MODE-MATCHING REFERENCE

Cases A6, A7 and A8 use a mode-matching solution of a centred iris as their reference. An iris of thickness t is a short section of reduced guide (opening w) between two steps. Each step is solved exactly in terms of waveguide modes, and the iris is the cascade step → section → mirrored step. With an increasing number of modes the solution converges to the exact result. For a centred iris only modes that are symmetric about the guide centre are excited.

Modes

Both iris types reduce to the same scalar problem in one transverse coordinate, with the following orthonormal mode functions, propagation constants and mode admittances:

TABLE 21 Mode sets of the two iris types. In the opening the same functions apply with w in place of the guide dimension, centred at the offset $c = (a - w)/2$ or $(b - w)/2$.

	Inductive iris (H-plane)	Capacitive iris (E-plane)
Opening	width w along the broad wall a	height w along the narrow wall b
Coupled modes	TE_{n0}, n odd	LSE_{1n}, n even (incl. TE_{10})
Mode function	$\varphi_n = \sqrt{2/a}\sin(n\pi x/a)$	$\varphi_n = \sqrt{(2-\delta_{n0})/b}\cos(n\pi y/b)$
Propagation	$\gamma_n^2 = (n\pi/a)^2 - k_0^2$	$\gamma_n^2 = (\pi/a)^2 + (n\pi/b)^2 - k_0^2$
Mode admittance	$Y_n \propto \gamma_n$	$Y_n \propto 1/\gamma_n$

For the capacitive iris the modes follow from the potential $\psi = \sin(\pi x/a)\cos(n\pi y/b)e^{-\gamma z}$. The transverse $\boldsymbol{H}$ follows from ψ without a z-derivative, which is why the admittance is proportional to $1/\gamma_n$.

Single step

The coupling between guide mode m and aperture mode n is evaluated in closed form. For the inductive iris, with $p = m\pi/a$ and $q = n\pi/w$:

$$X_{mn} = \frac{1}{\sqrt{aw}}\left[\frac{\sin((p-q)w + pc) - \sin(pc)}{p-q} - \frac{\sin((p+q)w + pc) - \sin(pc)}{p+q}\right], \tag{10}$$

where the first fraction becomes $w\cos(pc)$ for $p = q$. The capacitive case is the same with cosines, so the second fraction is added instead of subtracted. Matching $\boldsymbol{E}$ over the full guide cross-section (it vanishes on the metal) and $\boldsymbol{H}$ over the opening gives the generalised scattering matrix of the step. With $\boldsymbol{F} = (\boldsymbol{Y}_w + \boldsymbol{X}^T\boldsymbol{Y}_a\boldsymbol{X})^{-1}$:

$$\boldsymbol{S}_{21} = 2\boldsymbol{F}\boldsymbol{X}^T\boldsymbol{Y}_a, \quad \boldsymbol{S}_{22} = \boldsymbol{F}(\boldsymbol{Y}_w - \boldsymbol{X}^T\boldsymbol{Y}_a\boldsymbol{X}), \quad \boldsymbol{S}_{11} = \boldsymbol{X}\boldsymbol{S}_{21} - \boldsymbol{I}, \quad \boldsymbol{S}_{12} = \boldsymbol{X}(\boldsymbol{S}_{22} + \boldsymbol{I}). \tag{11}$$

Iris and verification

The section of length t is $\boldsymbol{P} = \mathrm{diag}(e^{-\gamma_n t})$, and the iris is the Redheffer star product of step, section and mirrored step. The S-parameters are the fundamental-mode entries, referred to the two faces of the iris. The number of aperture modes is kept at w/a (or w/b) times the number of guide modes. At zero thickness this ratio is required for convergence to the correct value; at finite thickness the result hardly depends on it.

The implementation was verified as follows:

- the closed-form coupling integrals agree with symbolic integration to 10^{-14};
- power is conserved, $|S_{11}|^2 + |S_{21}|^2 = 1$, to 10^{-15};

- between 320 and 1280 modes the S-parameters change by about 10^{-6}; the reference uses 200;
- at zero thickness the solution is exactly a shunt element, $S_{21} = 2/(2 \pm jB)$, to 10^{-15};
- at zero thickness EMerge (A7) and mode matching agree to better than −70 dB, two independent methods.